Cambright Solved Paper

Tags	2023 Additional Math CIE IGCSE May/June P1 V2
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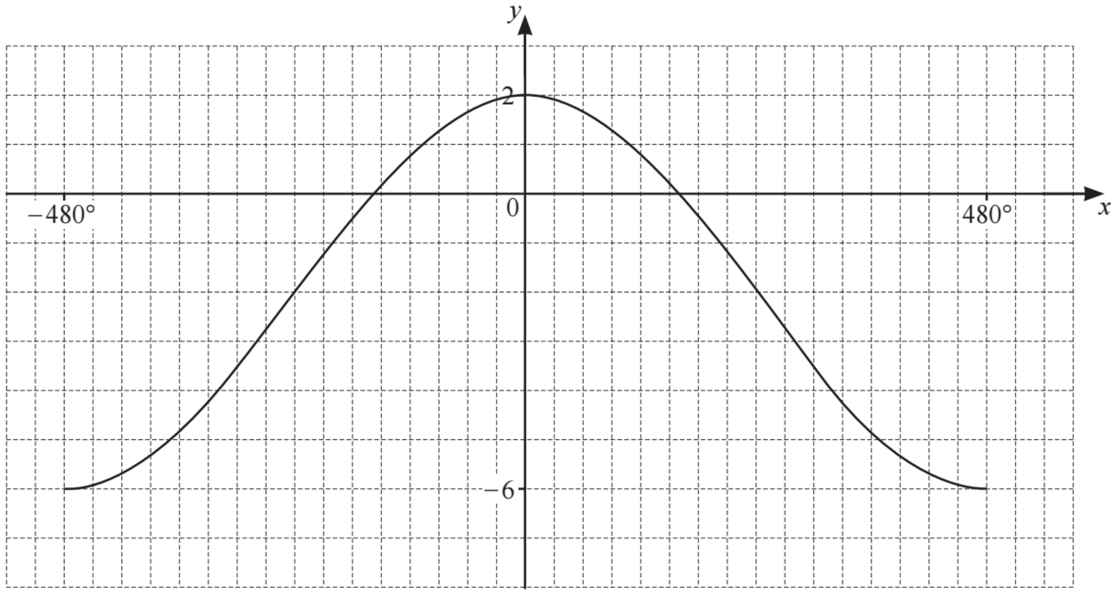
Status	Done

1 The diagram shows the graph of $y = a\cos bx + c$. Find the values of the constants a, b and c. [3]



$$a = 4(amplitude\ from\ center\ line)$$
$$period = \frac{360^\circ}{b}$$
$$960^\circ = \frac{360^\circ}{b}$$
$$b = \frac{360^\circ}{960^\circ} = \frac{3}{8}$$
$$c = -2(center\ line)$$

2 **DO NOT USE A CALCULATOR IN THIS QUESTION.**

Solve the equation $(2+\sqrt{5})x^2 = 4x + 3(2-\sqrt{5})$, giving your answers in the form $a + b\sqrt{5}$ where a and b are integers. [5]

$(2+\sqrt{5})x^2 - 4x - 3(2-\sqrt{5}) = 0$

Using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(2+\sqrt{5}) - 3(2-\sqrt{5})}}{2(2+\sqrt{5})}$$

$$x = \frac{4 \pm \sqrt{16 + 12(4-5)}}{2(2+\sqrt{5})}$$

$$x = \frac{4 \pm \sqrt{16 - 12}}{2(2+\sqrt{5})}$$

$$x = \frac{4 \pm \sqrt{4}}{2(2+\sqrt{5})}$$

$$x = \frac{4 \pm 2}{2(2+\sqrt{5})}$$

$$x = \frac{2 \pm 1}{2+\sqrt{5}}$$

$$x = \frac{2+1}{2+\sqrt{5}} \quad or \quad x = \frac{2-1}{2+\sqrt{5}}$$

$$x = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$x = \frac{6-3\sqrt{5}}{4-5}$$

$$x = -6 + 3\sqrt{5}$$

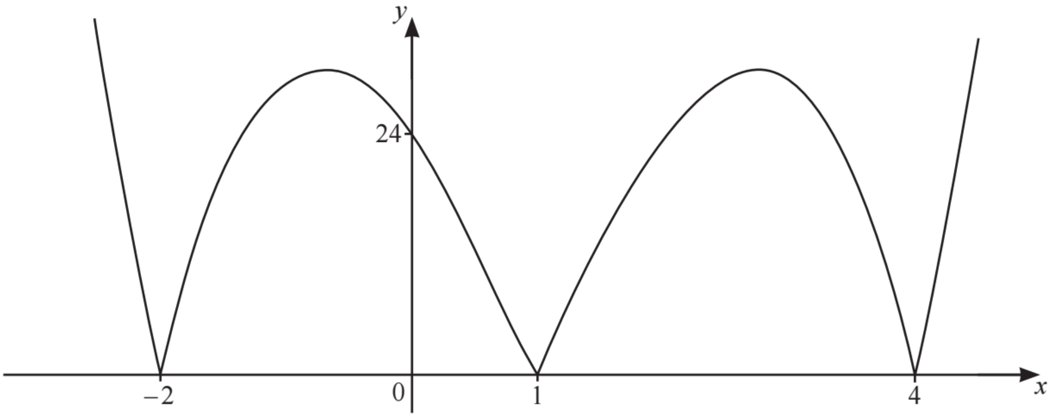
$$Or$$

$$x = \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$x = \frac{2-\sqrt{5}}{4-5}$$

$$x = -2 + \sqrt{5}$$

3 **(a)**



The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a cubic polynomial. Find, in factorised form, the possible expressions for $f(x)$. [3]

The 3 roots where y = 0 are x = -2, x = 1, and x = 4.

Therefore $f(x) = (x+2)(x-1)(x-4)$

When x = 0, y = ±24 because it is a modulus graph

$y = k \times f(x)$

$y = k \times (x+2)(x-1)(x-4)$

$\pm 24 = k \times (0+2)(0-1)(0-4)$

$\pm 24 = k \times (2)(-1)(-4)$

$k = \dfrac{\pm 24}{-8}$

$k = \pm 3$

Therefore final answer: $f(x) = \pm 3(x+2)(x-1)(x-4)$

(b) Solve the inequality $|5x-2| \leqslant |4x+1|$. [4]

First, square both sides to remove the modulus

$(5x-2)^2 \leq (4x+1)^2$

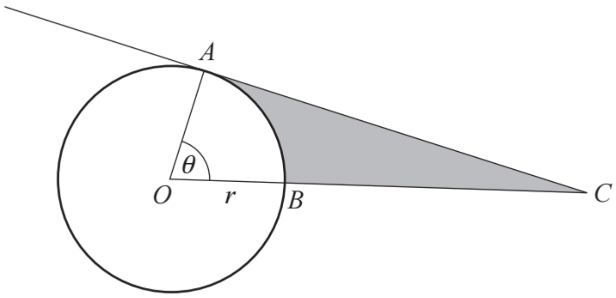
$25x^2 - 20x + 4 \leq 16x^2 + 8x + 1$

$9x^2 - 28x + 3 \leq 0$

$(9x-1)(x-3) \leq 0$

$\dfrac{1}{9} \leq x \leq 3$

4 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a circle with centre O and radius r. The points A and B lie on the circumference of the circle such that the angle AOB is θ and the length of the minor arc AB is 12. The area of the minor sector AOB is 57.6 cm^2. The point C lies on the tangent to the circle at A such that OBC is a straight line.

(a) Find the values of r and θ. [4]

First, don't forget to change your calculator to radians!!

Since the length of minor arc AB = 12,

$$r\theta = 12$$

$$\theta = \frac{12}{r}$$

Since the area of the minor sector is 57.6 cm²,

$$\frac{1}{2}r^2\theta = 57.6$$

$$r^2 \times \frac{12}{r} = 115.2$$

$$r = 9.6$$

Sub r = 9.6 in $\theta = \frac{12}{r}$

$$\theta = \frac{12}{9.6}$$

$$\theta = 1.25$$

(b) Find the area of the shaded region. Give your answer correct to 1 decimal place. [3]

To find the shaded area, we can find the triangle's area and subtract the sector's area(already given).

The area of a triangle is $\frac{1}{2}bh$, where b is AC and h is OA.

To find AC, $tan\theta = \frac{AC}{OA}$

$tan1.25 = \frac{AC}{9.6}$

$AC = 28.89$

Area of triangle = $\frac{1}{2}bh$

$\frac{1}{2} \times 28.89 \times 9.6 = 138.672cm^2$

$Shaded\ area = Triangle\ area - sector\ area$

$Shaded\ area = 138.672 - 57.6$

$Shaded\ area = 81.072$

3 significant figures = 81.1 cm^2

5 **(a)** Find the exact solutions of the equation $6p^{\frac{1}{3}} - 5p^{-\frac{1}{3}} - 13 = 0$. [4]

let $x = p^{\frac{1}{3}}$

$6x - 5x^{-1} - 13 = 0$

Multiply both sides by x

$6x^2 - 13x - 5 = 0$

$(2x - 5)(3x + 1) = 0$

$x = \frac{5}{2}\ \ or\ \ x = -\frac{1}{3}$

$p^{\frac{1}{3}} = \frac{5}{2}\ \ or\ \ p^{\frac{1}{3}} = -\frac{1}{3}$

$p = \frac{125}{8}\ \ or\ \ p = -\frac{1}{27}$

(b) Solve the equation $2\lg(2x+5)-\lg(x+2)=1$, giving your answers in exact form. [6]

Using log rules, we can move the coefficient 2 into a square

$lg(2x+5)^2 - lg(x+2) = 1$

When 2 logs of the same base are subtracted, we can divide them. 1 is the same as lg 10.

$lg\frac{(2x+5)^2}{x+2} = lg10$

Remove the log from both sides

$\frac{(2x+5)^2}{x+2} = 10$

$4x^2 + 20x + 25 = 10x + 20$

$4x^2 + 10x + 5 = 0$

Using quadratic formula:

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-10\pm\sqrt{10^2-4(4)(5)}}{2(4)}$

$x = \frac{-10\pm\sqrt{100-80}}{8}$

$x = \frac{-10\pm\sqrt{20}}{8}$

$x = \frac{-10\pm2\sqrt{5}}{8}$

$x = \frac{-5\pm\sqrt{5}}{4}$

$x = \frac{-5+\sqrt{5}}{4}$ or $x = \frac{-5-\sqrt{5}}{4}$

6 **(a)** Given that $\cot^2\theta = \frac{1}{y+2}$ and $\sec\theta = x-4$, find y in terms of x. [2]

We can use $sec^2\theta = 1 + tan^2\theta$ and $tan\theta = \frac{1}{cot\theta}$

$\frac{1}{cot^2\theta} = y + 2$, $sec^2\theta = (x-4)^2$

$$sec^2\theta = 1 + tan^2\theta$$

$$sec^2\theta = 1 + \frac{1}{cot^2\theta}$$

$$(x-4)^2 = 1 + y + 2$$

$$y + 3 = x^2 - 8x + 16$$

$$y = x^2 - 8x + 13$$

(b) Solve the equation $\sqrt{3}\operatorname{cosec}\left(2\phi + \frac{3\pi}{4}\right) = 2$, for $-\pi < \phi < \pi$, giving your answers in terms of π. [5]

$$cosec(2\phi + \frac{3\pi}{4}) = \frac{2}{\sqrt{3}}$$

$$sin(2\phi + \frac{3\pi}{4}) = \frac{\sqrt{3}}{2}$$

let $2\phi + \frac{3\pi}{4} = x$

$$2\phi = x - \frac{3\pi}{4}$$

$$\phi = \frac{x}{2} - \frac{3\pi}{8}$$

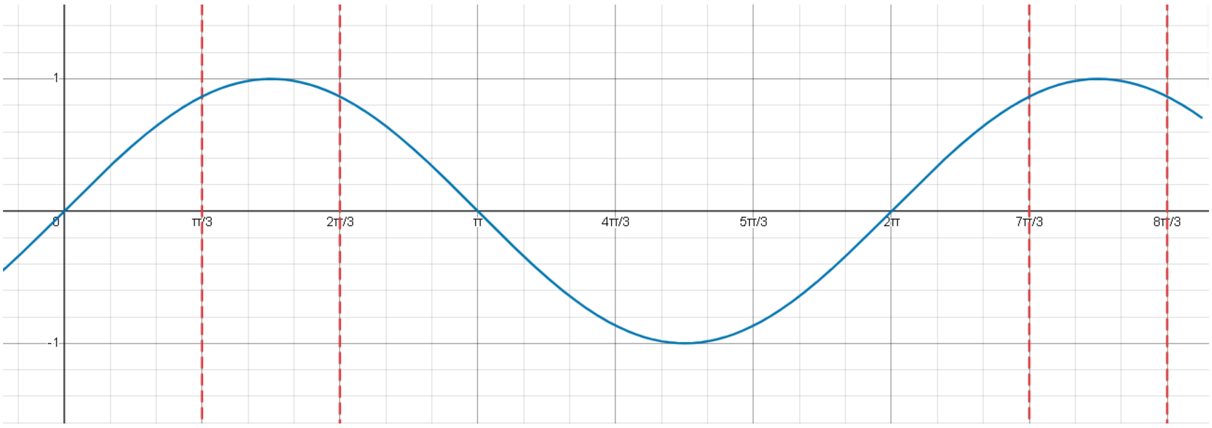
$$-\pi < \frac{x}{2} - \frac{3\pi}{8} < \pi$$

$$-\pi + \frac{3\pi}{8} < \frac{x}{2} < \pi + \frac{3\pi}{8}$$

$$-\frac{5\pi}{8} < \frac{x}{2} < \frac{11\pi}{8}$$

$$-\frac{5\pi}{4} < x < \frac{11\pi}{4}$$

Now, we can sketch the graph of $sin\ x = \frac{\sqrt{3}}{2}$



https://www.desmos.com/calculator/oc5thczxlb

Here we can see 4 solutions

They are $x = \frac{\pi}{3}, x = \frac{2\pi}{3}, x = \frac{7\pi}{3}, and\ x = \frac{8\pi}{3}$

Now substitute these values in $\phi = \frac{x}{2} - \frac{3\pi}{8}$ for our answers and we get

$\phi = -\frac{5\pi}{24},\ \phi = -\frac{\pi}{24},\ \phi = -\frac{19\pi}{24},\ \phi = \frac{23\pi}{24}$

7 (a) Find the number of ways in which 14 people can be put into 4 groups containing 2, 3, 4 and 5 people. [3]

$14C2 \times 12C3 \times 9C4 \times 5C5 = 2,522,520$

(b) 6-digit numbers are to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Each digit may be used only once in any 6-digit number. A 6-digit number must not start with 0. Find how many 6-digit numbers can be formed if

(i) there are no further restrictions [1]

_ _ _ _ _ _ This is the 6 digits

The first digit can be 1 - 9, so there are 9 possibilities

The second to sixth digits can be 0 - 9 with one number already used in the first digit, so 9 possibilities

To arrange 5 digits out of 9 possibilities, $9P5 = 15120$

First digit can be 9 numbers, $9 \times 15120 = 136080$

(ii) the 6-digit number is divisible by 10 [1]

The final digit must be 0 so it is divisible by 10, so the remaining 5 digits can be any number in 1 - 9

$9P5 = 15120$

(iii) the 6-digit number is greater than 500 000 and even. [3]

Case 1: If the first digit is odd, the last digit must be 0, 2, 4, 6, or 8 so it is even (5 possibilities)

Case 2: If the first digit is even, the last digit will be one of the remaining even numbers (4 possibilities)

Let's consider case 1 first; the first digit will be an odd number greater than or equal to 5(5, 7, 9) so there are 3 possibilities. The final digit will be an even number and has 5 possibilities. The 4 middle digits can be anything remaining so 8 possibilities.

So $3 \times 5 \times 8P4 = 25200$

Now for case 2; the first digit will be even and greater than 5(6, 8) so 2 possibilities. Final digit will be a remaining even number so 4 possibilities. The 4 middle digits can be any number left so 8 possibilities.

So $2 \times 4 \times 8P4 = 13440$

So $total \ = 25200 + 13440 = 38640$

8 It is given that $f(x) = 2\ln(3x-4)$ for $x > a$.

(a) Write down the least possible value of a. [1]

There cannot be 0 or a negative number within a log, so $a = \frac{4}{3}$

(b) Write down the range of f. [1]

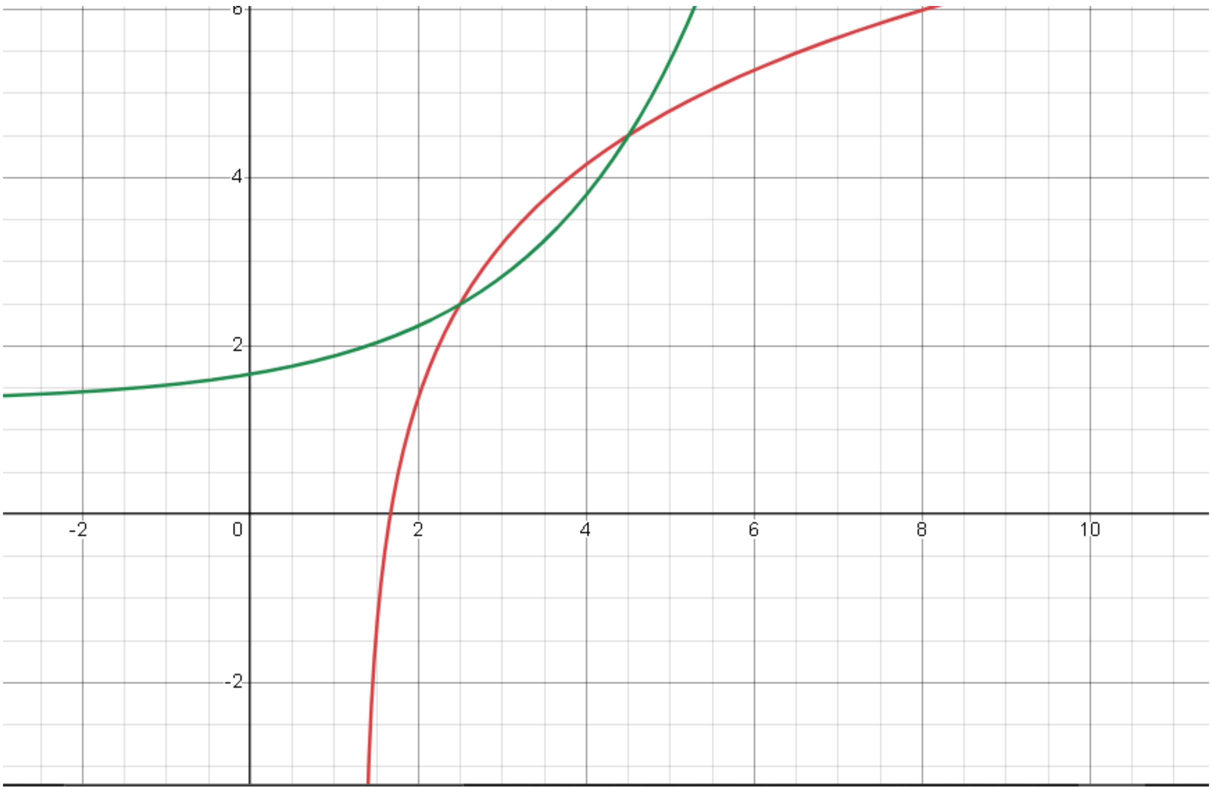
Any input of $x > \frac{4}{3}$ will output an infinite amount of numbers, so $f(x) \in \mathbb{R}$

(c) It is given that the equation $f(x) = f^{-1}(x)$ has two solutions. (You do not need to solve this equation). Using your answer to **part (a)**, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes below, stating the coordinates of the points where the graphs meet the axes. [4]

For $f(x)$, there will be an asymptote where $x = \frac{4}{3}$, so for $f^{-1}(x)$, there will be an asymptote where $y = \frac{4}{3}$.

When $f(x) = 0$, we will get our x-intercept: $2ln(3x - 4) = 0$ leading to $x = \frac{5}{3}$, so when $f^{-1}(x) = 0$, we will get an y-intercept of $y = \frac{5}{3}$

Using these 4 points, we can graph as so:



https://www.desmos.com/calculator/pm0t14gayz

Make sure to point out the intercept points $(\frac{5}{3},\ 0)\ and\ (0,\ \frac{5}{3})$

It is given that $g(x) = 2x - 3$ for $x \geqslant 3$.

(d) **(i)** Find an expression for $g(g(x))$. [1]

$$g(g(x)) = g(2x - 3)$$
$$= 2(2x - 3) - 3$$

$= 4x - 6 - 3$
$= 4x - 9$

(ii) Hence solve the equation $fg(g(x)) = 4$ giving your answer in exact form. [3]

$f(x) = 2ln(3x - 4)$

$fg(g(x)) = 4$

$f(4x - 9) = 4$

$2ln(3(4x - 9) - 4) = 4$

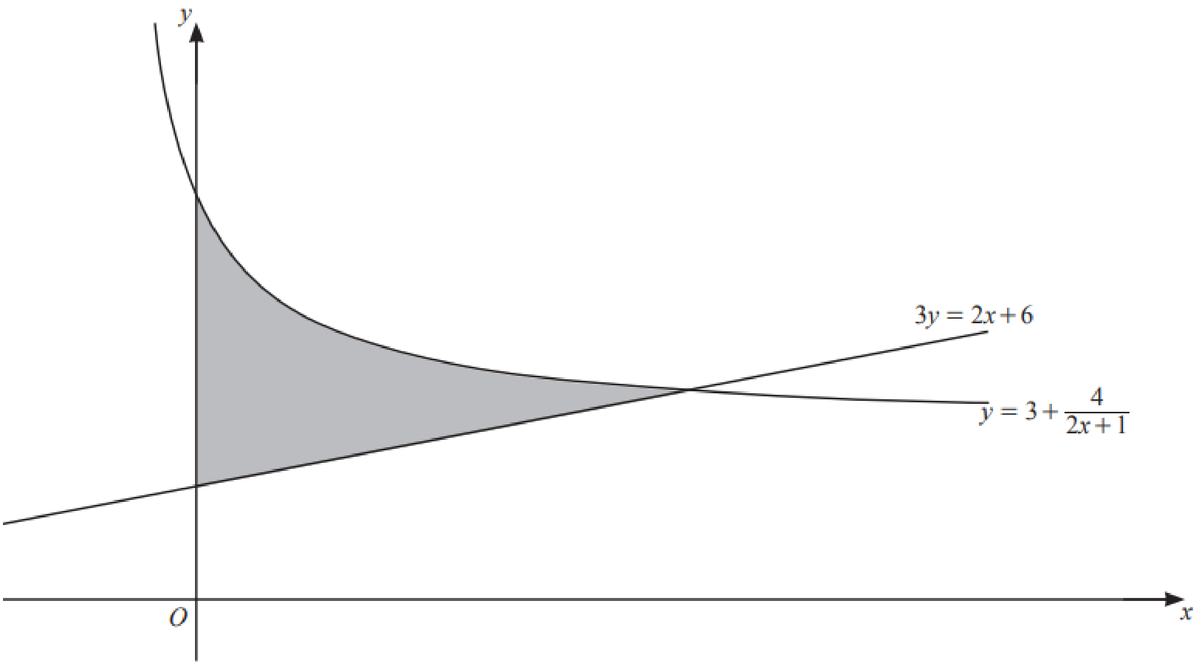
$ln(12x - 27 - 4) = 2$

$12x - 31 = e^2$

$12x = e^2 + 31$

$x = \frac{e^2+31}{12}$

9



The diagram shows part of the curve $y = 3 + \frac{4}{2x+1}$ and the straight line $3y = 2x + 6$. Find the area of the shaded region, giving your answer in exact form. [10]

$3y = 2x + 6$

$y = \frac{2x+6}{3}$

Find the intersection point of the 2 functions:

$\frac{2x+6}{3} = 3 + \frac{4}{2x+1}$

$2x + 6 = 9 + \frac{12}{2x+1}$

$2x - 3 = \frac{12}{2x+1}$

$4x^2 - 4x - 3 = 12$

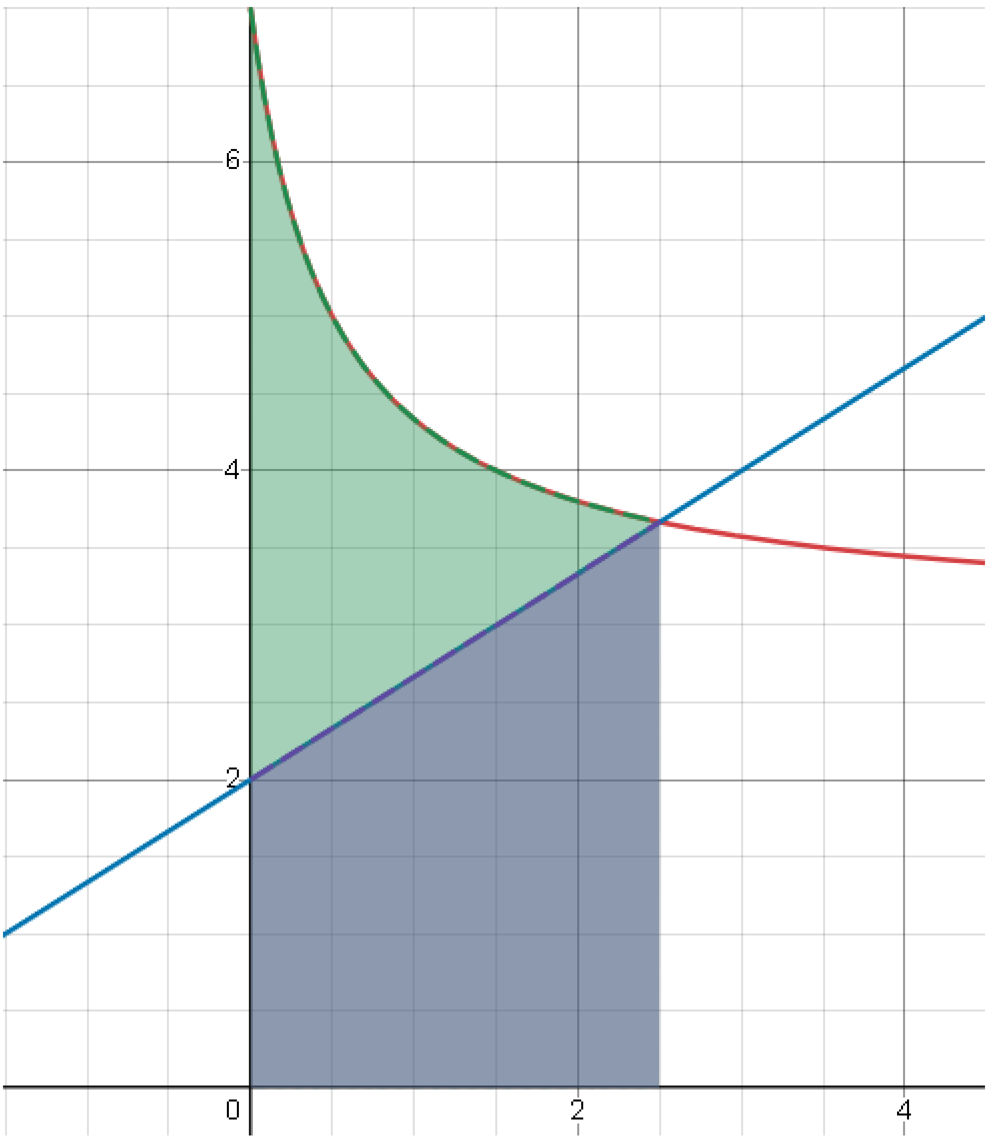
$4x^2 - 4x - 15 = 0$

$x = \frac{5}{2} \;\; or \;\; x = -\frac{3}{2}$

Here, the intersection point is obviously positive, so $x = \frac{5}{2}$

In the straight line, when $x = \frac{5}{2}$, $y = \frac{11}{3}$

To find the area, we can find the curve area - the line area



https://www.desmos.com/calculator/farqvrgza2

First let's find the integral of the curve

$$\int 3 + \frac{4}{2x+1} dx = 3x + 4\int \frac{1}{2x+1} dx = 3x + (4 \times \frac{1}{2} \times ln(2x+1)) =$$
$$3x + 2ln(2x+1)$$

$$Area \ = \int_0^{\frac{5}{2}} 3 + \frac{4}{2x+1} dx - (\frac{2+11/3}{2} \times \frac{5}{2})$$

$$Area \ = [3x + 2ln(2x+1)]_0^{\frac{5}{2}} - \frac{85}{12}$$

$$Area \ = [3 \times \frac{5}{2} + 2ln(2 \times \frac{5}{2} + 1)] - [3 \times 0 + 2ln(2 \times 0 + 1)] - \frac{85}{12}$$

$$Area = \frac{15}{2} + 2ln(6) - 0 - \frac{85}{12}$$

$$Area = \frac{5}{12} + 2ln(6)$$

10 **(a)** The first three terms of an arithmetic progression are $(2x+1)$, $4(2x+1)$ and $7(2x+1)$, where $x \neq -\frac{1}{2}$.

(i) Show that the sum to n terms can be written in the form $\frac{n}{2}(2x+1)(An+B)$, where A and B are integers to be found. [2]

common difference $d = 4(2x+1) - (2x+1) = 3(2x+1)$

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{n}{2}\{2(2x+1) + (n-1)3(2x+1)\}$$

Factor out (2x + 1)

$$= \frac{n}{2}(2 + 3n - 1)(2x+1)$$

$$= \frac{n}{2}(3n+1)(2x+1)$$

(ii) Given that the sum to n terms is $(54n+37)(2x+1)$, find the value of n. [2]

$$\frac{n}{2}(3n+1)(2x+1) = (54n+37)(2x+1)$$

$$n(3n+1) = 2(54n+37)$$

$$3n^2 + n = 108n + 74$$

$$3n^2 - 109n - 37 = 0$$

If you use your calculator, you will find that n = 36.669 or n = -0.336

So we take n = 37 since it is the closest possible answer

(iii) Given also that the sum to n terms in **part (ii)** is equal to 1017.5, find the value of x. [2]

$$(54n+37)(2x+1) = 1017.5$$

$$(54 \times 37 + 37)(2x+1) = 1017.5$$

$$2035(2x+1) = 1017.5$$

$$2x + 1 = \frac{1}{2}$$

$2x = -\frac{1}{2}$

$x = -\frac{1}{4}$

(b) The first three terms of a geometric progression are $(2y+1)$, $3(2y+1)^2$ and $9(2y+1)^3$, where $y \neq -\frac{1}{2}$.

Given that the nth term of the progression is equal to 4 times the $(n+2)$th term, find the possible values of y, giving your answers as fractions. [4]

Common ratio $r = \frac{3(2y+1)^2}{2y+1} = 3(2y+1)$

$n^{th}\ term = ar^{n-1} = (2y+1)[3(2y+1)]^{n-1} = (2y+1) \times 3^{n-1} \times (2y+1)^{n-1}$

$= (2y+1)^n \times 3^{n-1}$

$(n+2)^{th}\ term = ar^{n+2-1} = (2y+1)[3(2y+1)]^{n+1} = (2y+1) \times 3^{n+1}(2y+1)^{n+1}$

$(2y+1)^{n+2} \times 3^{n+1}$

$n^{th}\ term = 4(n+2)^{th}\ term$

$(2y+1)^n \times 3^{n-1} = 4(2y+1)^{n+2} \times 3^{n+1}$

$3^{n-1} \div 3^{n+1} = 4(2y+1)^{n+2} \div (2y+1)^n$

$3^{-2} = 4(2y+1)^2$

$\frac{1}{9} \div 4 = (2y+1)^2$

$2y+1 = \pm\sqrt{\frac{1}{36}}$

$2y+1 = \pm\frac{1}{6}$

$2y = -1 \pm \frac{1}{6}$

$y = -\frac{5}{6} \div 2 \ \ or \ \ y = -\frac{7}{6} \div 2$

$y = -\frac{5}{12} \ \ or \ \ y = -\frac{7}{12}$

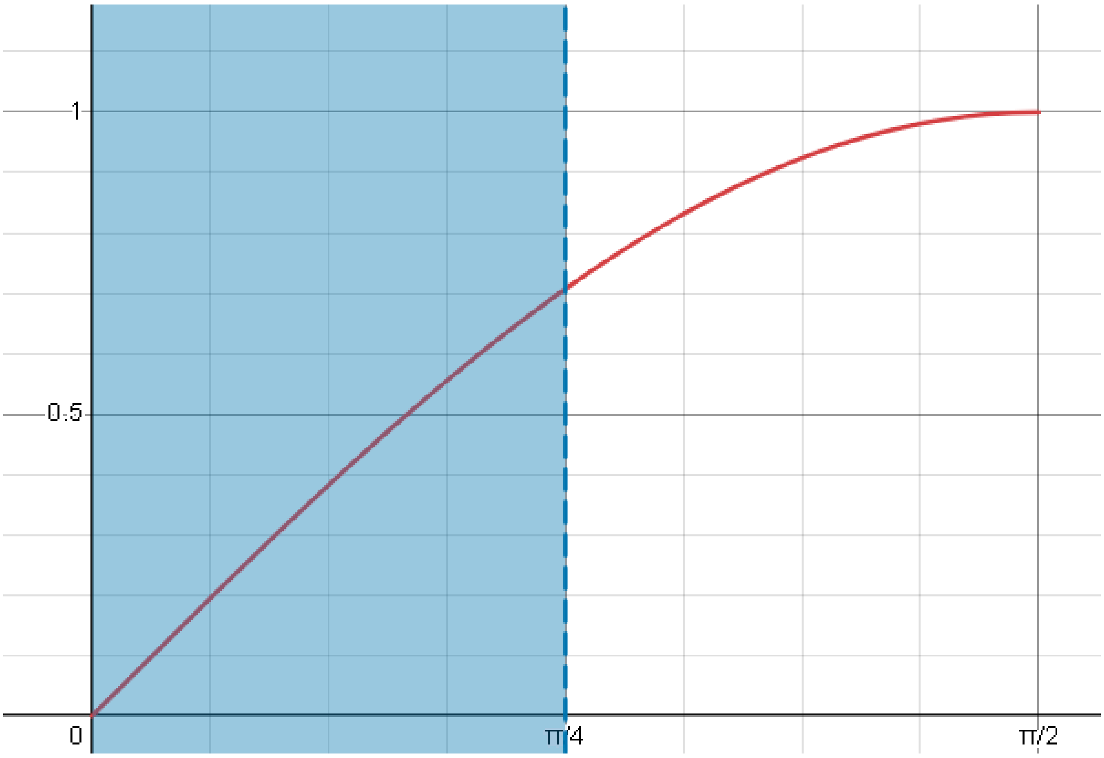
(c) The first three terms of a different geometric progression are $\sin\theta$, $2\sin^3\theta$ and $4\sin^5\theta$, for $0 < \theta < \frac{\pi}{2}$. Find the values of θ for which the progression has a sum to infinity. [3]

common ratio $r = \dfrac{2sin^3\theta}{sin\theta} = 2sin^2\theta$

For sum to infinity: $0 < 2sin^2\theta < 1$

$sin^2\theta < \dfrac{1}{2}$

$sin\theta = \pm\dfrac{1}{\sqrt{2}}$



https://www.desmos.com/calculator/trcerzrx9g

$0 < \theta < \dfrac{\pi}{4}$

Additional notes

Websites and resources used:

- [Desmos graphing calculator]

If you find any errors or mistakes within this paper, please contact us and we will fix them as soon as possible.